ΔΙΑΓΩΝΙΣΜΑ ΤΑΛΑΝΤΩΣΕΙΣ ΙΑΝΟΥΑΡΙΟΣ 2025

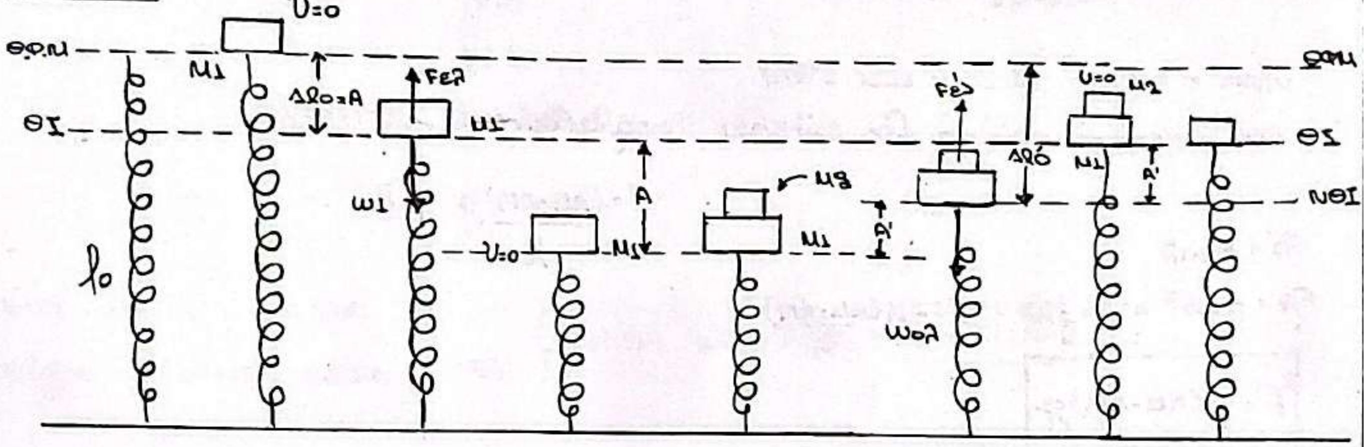
ΘΕΜΑ Α

$A1 \rightarrow \gamma \quad A2 \rightarrow \delta \quad A3 \rightarrow \beta \quad A4 \rightarrow \beta$

$A5 \rightarrow$ α-Σ, β-Λ, γ-Σ, δ-Σ, ε-Σ

ΘΕΜΑ Β

(Β1)



Β1α| ΘΙ: $\Sigma F=0 \Rightarrow F_{\varepsilon\lambda} = w_1 \Rightarrow k\Delta l_0 = m_1 g = 2mg$

$\Delta l_0 = 2mg/k$ και $A = 2mg/k$ (1)

ΝΘΙ: $\Sigma F=0 \Rightarrow F'_{\varepsilon\lambda} = w_{o\lambda} \Rightarrow k\Delta l'_0 = (m_1+m_2)g \Rightarrow$

$\Delta l'_0 = \dfrac{(2m+m)g}{k} \Rightarrow \Delta l'_0 = \dfrac{3mg}{k}$ (2)

$A' = \Delta l'_0 - \Delta l_0 = 3mg/k - 2mg/k = mg/k$ (3)

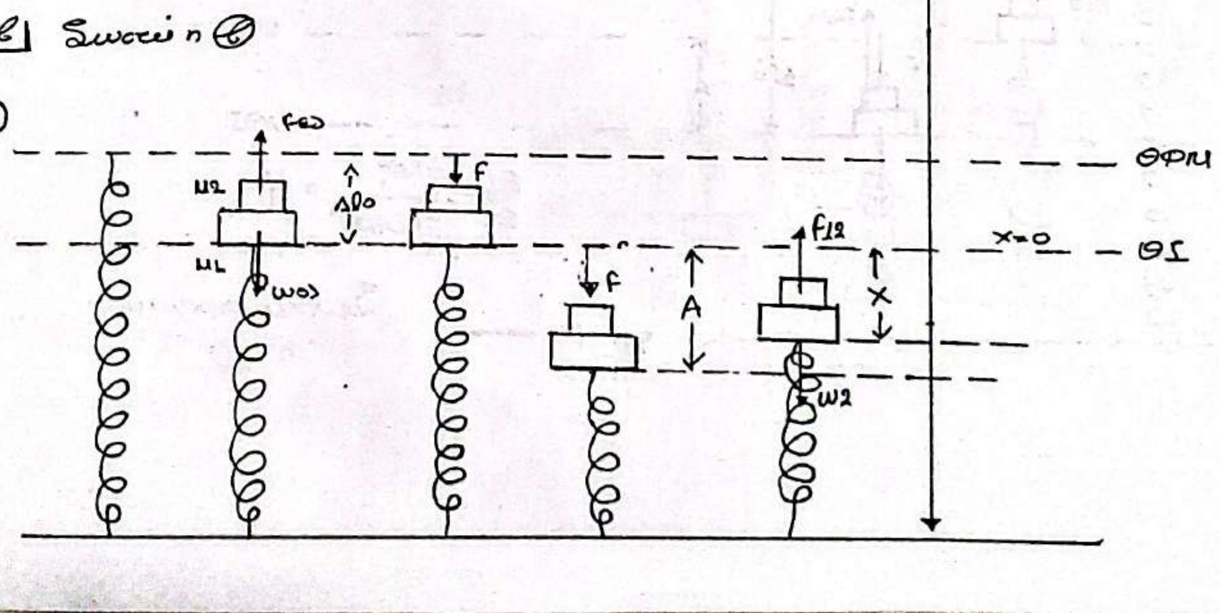
$S_1 = 2A = 2 \cdot 2mg/k = 4mg/k$ (4)

$S_2 = 2A' = 2mg/k$ (5)

(4),(5) $\dfrac{S_1}{S_2} = 2$ (6) Σωστή επιλογή η (α)

Β1β| Σωστή η (β)

(Β2)



(ΘΙ) $\Sigma F = 0 \Rightarrow K\Delta l_0 = (m_1+m_2)g$ (1)

$\Sigma F_2 = -D_2 x \Rightarrow w_2 - F_{12} = -D_2 x$ (2)

$w_2 + D_2 x = F_{12}$ Η επαφή χάνεται στη θέση όπου $F_{12} = 0 \Rightarrow w_2 + D_2 x = 0$

$x = -\dfrac{w_2}{D_2}$ αλλά (με απόδειξη) $D_2 = \dfrac{m_2 K}{m_1+m_2}$

Άρα, $x = -\dfrac{m_2 g}{\frac{m_2 K}{m_1+m_2}} = -\dfrac{(m_1+m_2)g}{K} = -\Delta l_0$ (3)

Άρα η επαφή χάνεται στη ΘΦΜ

και η επαφή οριακά δεν χάνεται καταλαβαίνουμε ότι $A = \Delta l_0$

$A = \dfrac{(m_1+m_2)g}{K}$ (4)

$F_A = E_{ταλ}$

$F_A = \frac{1}{2}kA^2 \Rightarrow F = \frac{1}{2}kA \Rightarrow F = \frac{1}{2}k\dfrac{(m_1+m_2)}{K}$

$\boxed{F = \frac{1}{2}(m_1+m_2)g}$

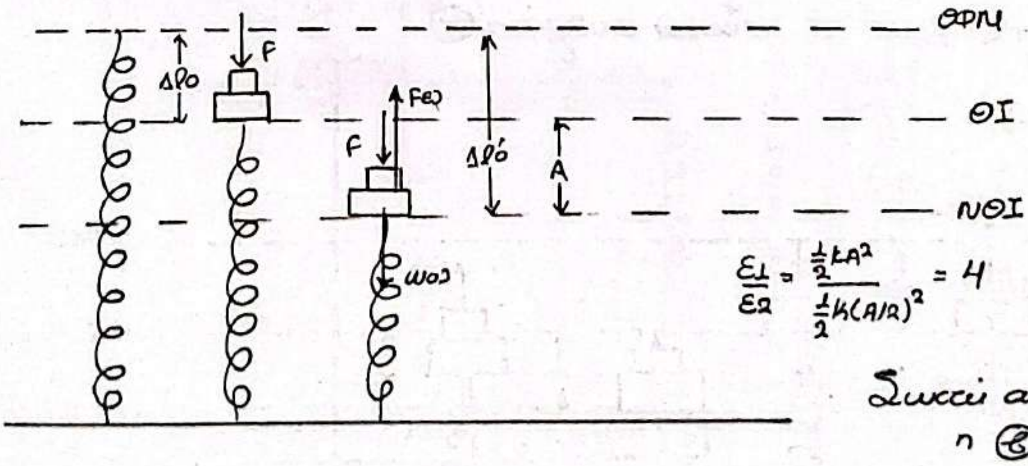
Όταν η F δρα μόνιμα και δεν αποσύρεται τότε η ταλάντωση γίνεται γύρω από τη ΝΘΙ στην οποία

$F + w_{ολ} = F_{ελ} \Rightarrow \frac{1}{2}(m_1+m_2)g + (m_1+m_2)g = k\Delta l_0'$

$\frac{3}{2}(m_1+m_2)g = k\Delta l_0' \Rightarrow \Delta l_0' = \frac{3}{2}\dfrac{(m_1+m_2)g}{k}$

Και το πλάτος της ταλάντωσης

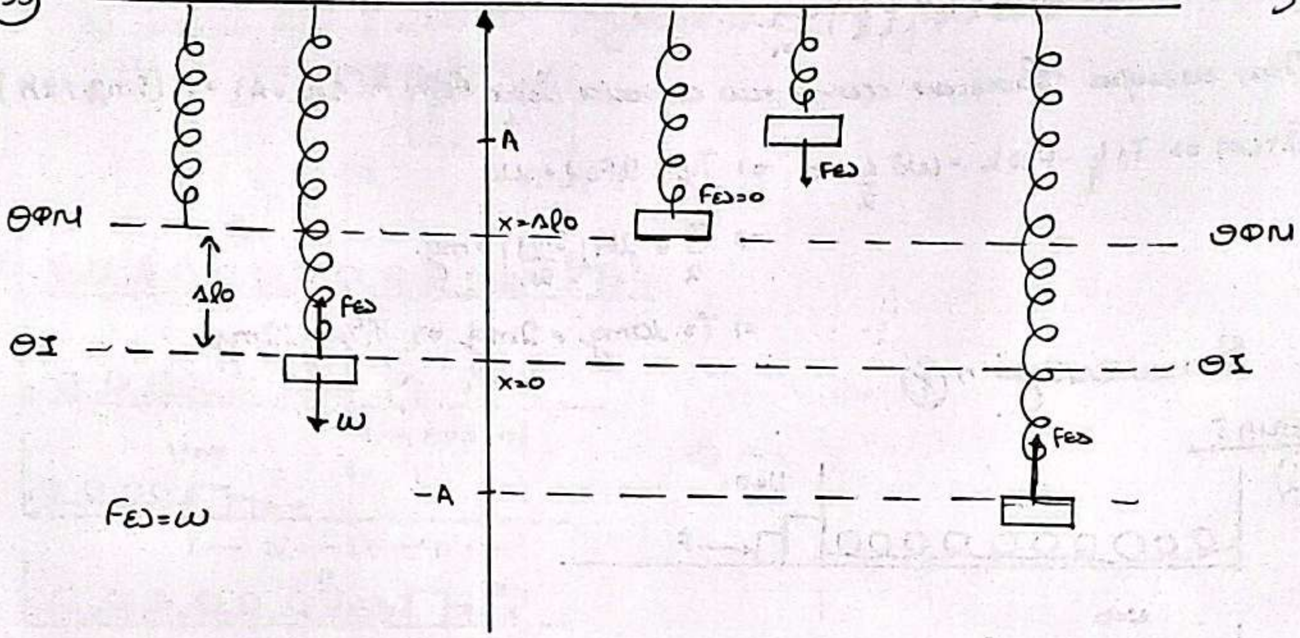
είναι $A' = \Delta l_0' - \Delta l_0 = \frac{1}{2}\left(\dfrac{m_1+m_2}{k}\right)g = \dfrac{A}{2}$ (5)



$\dfrac{E_1}{E_2} = \dfrac{\frac{1}{2}kA^2}{\frac{1}{2}k(A/2)^2} = 4$

Σωστή απάντηση η (β)

(B3)



$x=0 \quad F_{ελ}=w \Rightarrow w=10N \quad (1)$

$x=\frac{1}{15}m \;,\; F_{ελ}=0 \;,\; \Delta l_0=\frac{1}{15} \quad (2)$

$\Big\}\ (1),(2) \quad k\Delta l_0 = 10 \Rightarrow \boxed{k=150N}$ ③

$x=A \quad F_{ελ}=-50N \;,\; -k\Delta l=-50$

$k\Delta l=50 \Rightarrow k(A-\Delta l_0)=50$

$A-\frac{1}{15}=\frac{50}{150} \Rightarrow A=\frac{1}{3}+\frac{1}{15} \Rightarrow A=\frac{5}{15}+\frac{1}{15}=\frac{6}{15}$

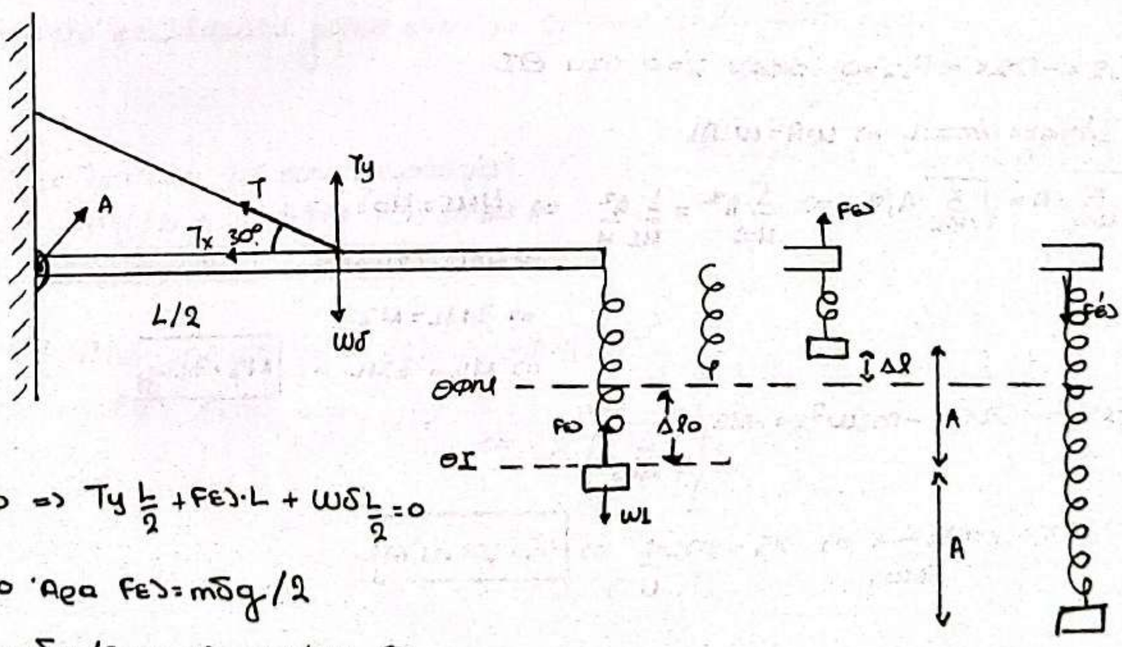
$\boxed{A=0{,}4m}$ ④

$E_{ταλ}=\frac{1}{2}kA^2=\frac{1}{2}150\cdot 0{,}16 \Rightarrow$

$E_{ταλ}=12J$

Σωστή επιλογή η (β)

(B4)



$Σ_{τ(A)}=0 \Rightarrow T_y\frac{L}{2}+F_{ελ}\cdot L+w_δ\frac{L}{2}=0$

$T_{ολ}=0$ Άρα $F_{ελ}=m_δ g/2$

$k\Delta l=m_δ g/2 \Rightarrow \Delta l=mg/2k$ ①

(1),(2) → $A = \Delta l_0 + \Delta l = \left(\frac{3}{2}\right)\frac{mg}{K}$

Όταν το σώμα βρίσκεται στην κάτω ακραία θέση $F_{ελ} = K(\Delta l_0 + A) = K(5mg/2K)$

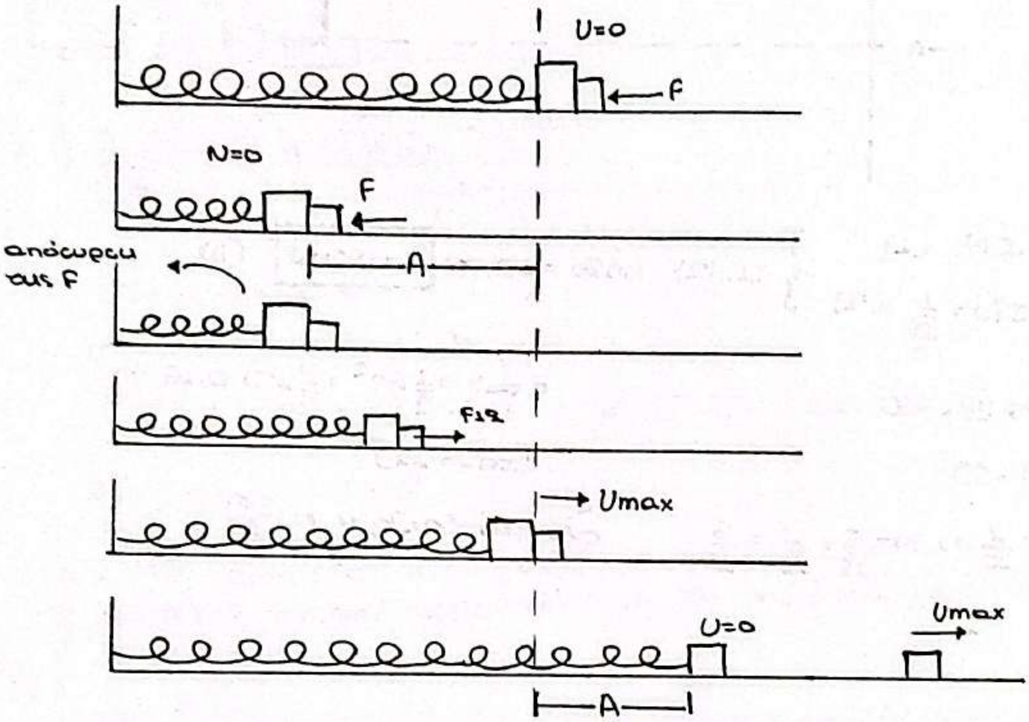
$\Sigma\tau_{(A)} \Rightarrow T_y \frac{L}{2} - F_{ελ} L - w\delta \frac{L}{2} = 0 \Rightarrow T_y = 2F_{ελ} + w\delta$

$\Rightarrow \frac{T}{2} = 2K\left(\frac{5mg}{2K}\right) + mg$

$\Rightarrow T = 10mg + 2mg \Rightarrow T_{ΘΡ} = 12mg$

Σωστή επιλογή η (δ)

ΘΕΜΑ Γ



(Γ1) $F_{12} = -D_2 x$ $\quad F_{12} = 0$ όταν $x = 0$ στη ΘΙ

$U_{max} = U_{max1} \Rightarrow \omega A = \omega_1 A_1$

$\sqrt{\frac{K}{M_{ολ}}} \cdot A = \sqrt{\frac{K}{M_1}}\, A/2 \Rightarrow \frac{1}{M_{ολ}} A^2 = \frac{1}{M_1}\frac{A^2}{4} \Rightarrow 4M_1 = M_{ολ}$

$\Rightarrow 4M_1 = M_1 + M_2$

$\Rightarrow 3M_1 = M_2$

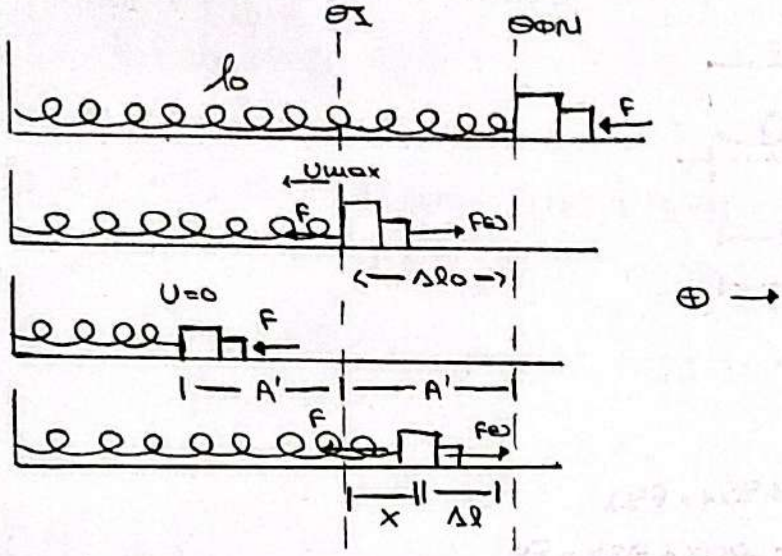
$\Rightarrow M_2 = 3M_1 \Rightarrow \boxed{M_2 = 3kg}$

(Γ2) $F_{12} = -D_2 x = -m_2\omega^2 x = -M_2\left(\sqrt{\frac{K}{M_{ολ}}}\right)^2 x \Rightarrow$

$-75x = -m_2 \frac{K}{M_{ολ}} x \Rightarrow 75 = m_2 \frac{K}{M_{ολ}} \Rightarrow \boxed{K = 100N/m}$

(Γ3) Για όση ώρα ασκείται η $\vec{F}$
το σύστημα των 2 σωμάτων πραγματοποιεί ΑΑΤ γύρω από
τη ΘΙ στην οποία $F = F_{ελ}$ =>

$$\boxed{F = k\Delta l_0}$$



$\Sigma F = -F + F_{ελ}$ =>

$-F + k(\Delta l) = -F + k(\Delta l_0 - x)$

$= \cancel{-F} + \cancel{k\Delta l_0} - kx = -kx$ => $D = k$ Άρα $υ_{max} = ωA' = \sqrt{\frac{k}{m_{ολ}}}\,A'$

$A' = 0,2m$ άρα $A = 2A'$

διότι η F αποσύρεται στην αριστερή ακραία θέση οπότε στη συνέχεια
η ταλάντωση γίνεται χωρίς την παρουσία της F και γύρω από τη ΘΦΜ

Άρα $A = 2A' = 0,4m$

και για την ταλάντωση γύρω από τη ΘΦΜ έχουμε $W_F = E_{ταλ}$ =>

$FA = \frac{1}{2}kA^2$ =>

$F = \frac{1}{2}kA$ => $F = \frac{1}{2}100 \cdot 0,4 = 20N$

$F = 20N$

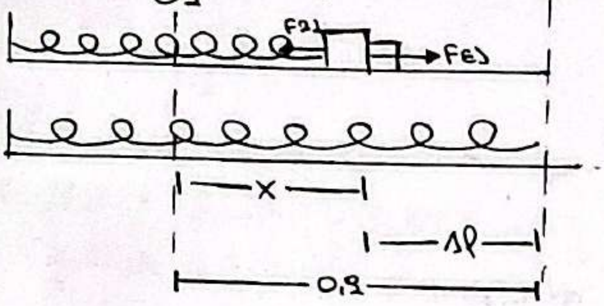
(Γ4) Η ταλάντωση με την παρουσία
της F αρχίζει στη ΘΦΜ που είναι η
δεξιά ακραία θέση της ταλάντωσης

Άρα, $x = A'ημ(ωt + φ_0) = 0,2ημ(5t + π/2)$ SI

καθώς $t=0$, $x=A$, $υ=0$

(Γ5) $F_{21} = -D_1 x = -m_1\omega^2 x \Rightarrow$

$$F_{21} = -1\left(\sqrt{\frac{k}{m_{ολ}}}\right)^2 x = -25x$$



$F_{ελ} - F_{21} = -25x \Rightarrow$

$F_{ελ} + 25x = F_{21} \Rightarrow$

$k\Delta l + 25x = F_{21} \Rightarrow k(0,2-x) + 25x = F_{21}$

$\Rightarrow 100 \cdot 0,2 - 100x + 25x = F_{21}$

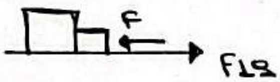
$\Rightarrow 20 - 75x = F_{21}$

F_{21max} όταν $x = -A'$

$F_{21max} = 20 - 75(-0,2) \Rightarrow$

$F_{21max} = 20 + 15 = 35N \Rightarrow \boxed{F_{21max} = 35N}$

ή $F_{21max} = F_{12max}$



$\Sigma F_2 = -D_2 x \Rightarrow$

$\Sigma F_2 = -75x \Rightarrow$

$F_{12} - F = -75x \Rightarrow F_{12} = F - 75x$

$\Rightarrow F_{12max} = 20 - 75(-0,2) = 35N$

ΘΕΜΑ Δ

Δ1) $\vec{P}_{ολ,αρχ} = \vec{P}_{ολ,τελ}$

$m_1 υ_1 = m_1 υ_1' + m_2 υ_2'$

$1 \cdot υ_1 = 1 \cdot υ_1' + 2 \cdot 2$

$υ_1 = υ_1' + 4$ (1)

$E_{μηχ,αρχ} = E_{μηχ,τελ}$

$K_{1,αρχ} + K_{2,αρχ} + U_{ελ,αρχ} = K_{1,τελ} + K_{2,τελ} + U_{ελ,τελ}$

$\frac{1}{2} m_1 υ_1^2 + \frac{1}{2} m_2 0^2 + 0 = \frac{1}{2} m_1 υ_1'^2 + \frac{1}{2} m_2 υ_2'^2 + \frac{1}{2} k \Delta\ell^2$

$υ_1^2 + 0 + 0 = υ_1'^2 + 2 \cdot 2^2 + 50 \cdot 0{,}4^2$

$υ_1^2 = υ_1'^2 + 8 + 8$

$υ_1^2 = υ_1'^2 + 16$ (2)

(1) (2) $\Rightarrow (υ_1' + 4)^2 = υ_1'^2 + 16$

$υ_1'^2 + 8υ_1' + \cancel{16} = υ_1'^2 + \cancel{16}$

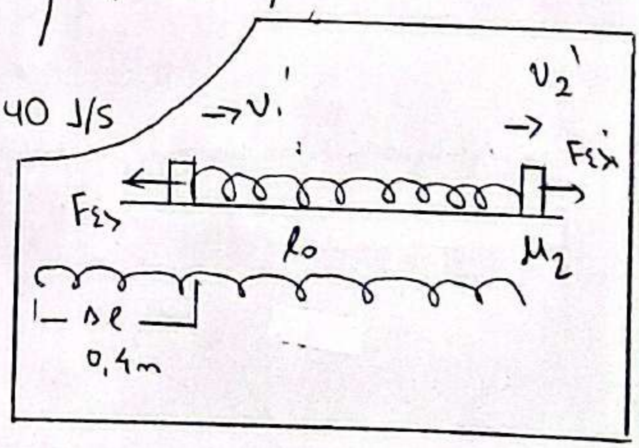
$8υ_1' = 0 \Rightarrow \boxed{υ_1' = 0}$ (3)

(1),(3) $\Rightarrow \boxed{υ_1 = 4 m/s}$ (4)

Δ2) $\left(\frac{dK}{dt}\right)_2 = \Sigma F_2 υ = F_{ελ} \cdot υ_2' = 50 \cdot 0{,}4 \cdot 2 = 40 \ J/s$ (5)

<u>α τρόπος</u> $\frac{dU_{ελ}}{dt} = \left(-\frac{dW_{Fελ1}}{dt}\right) + \left(-\frac{dW_{Fελ2}}{dt}\right) = -\left(-F_{ελ_1} \cdot υ_1'\right) - F_{ελ} υ_2'$

$= F_{ελ} \cdot \cancelto{0}{υ_1'} - F_{ελ} \cdot υ_2' \overset{(5)}{=} -40 \ J/s$



<u>β τρόπος</u> $E_{μηχ} = σταθερή$

$U_{ελ} + K_1 + K_2 = σταθερή$

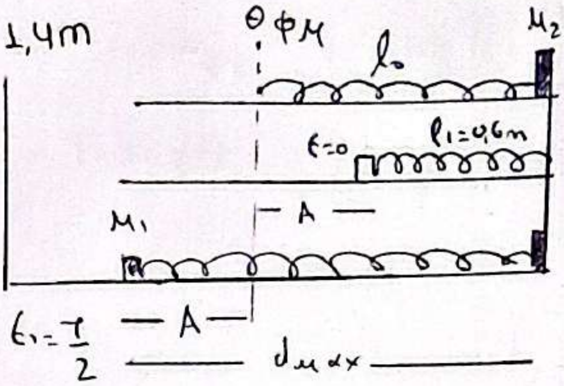
$\frac{dU_{ελ}}{dt} + \frac{dK_1}{dt} + \frac{dK_2}{dt} = 0$

$\frac{dU_{ελ}}{dt} = -\frac{dK_1}{dt} - \frac{dK_2}{dt} = -\cancelto{0}{\Sigma F_1 \cdot υ_1'} - \Sigma F υ_2' = -k\ell \cdot υ_2' = -40 \ J/s$

Δ3) Το σώμα Σ1 πραγματοποιεί Α.Α.Τ με πλάτος $A=0,4m$

$$d_{max} = \ell_1 + A + A = 0,6 + 0,4 + 0,4 = 1,4m$$

$$\Delta t = \frac{T}{2} = \frac{1}{2} 2\pi\sqrt{\frac{m_1}{k}} = 0,1\pi\sqrt{2}\ sec$$



Δ4) $$\Sigma F_3 = -D_3 x = -\frac{k m_3}{m_1+m_3} x$$

$$T_{\sigma\tau} = -\frac{K m_3}{m_1+m_3} x$$

$$|T_{\sigma\tau}| = \frac{50 m_3}{1+m_3}|x|$$

$$|T_{\sigma\tau,max}| = \frac{50 m_3}{1+m_3} A = \frac{20 m_3}{1+m_3}$$

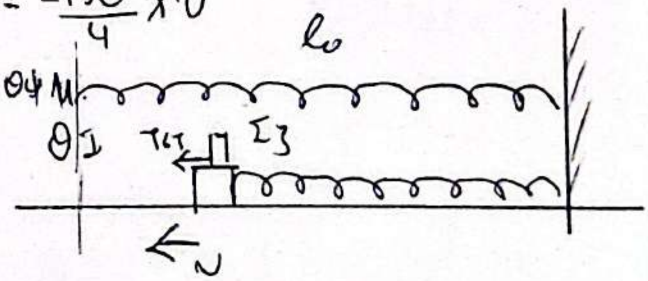
Πρέπει $$\frac{20 m_3}{1+m_3} \leq \mu_{\sigma\tau}\cdot N = \mu_{\sigma\tau} m_3 g$$

$$\frac{20}{1+m_3} \leq \mu_{\sigma\tau}\cdot g$$

$$\frac{20}{1+m_3} \leq \mu_{\sigma\tau}\cdot 10 = 0,5\cdot 10 = 5$$

$$\frac{20}{5} = 1 + m_3 \Rightarrow 4 = 1 + m_3 \Rightarrow \boxed{m_3 = 3kg}$$

Δ5) $$\frac{dW_{T\sigma\tau}}{dt} = T_{\sigma\tau}\cdot\upsilon = -\frac{50\cdot 3}{1+3} x\cdot\upsilon = -\frac{150}{4} x\cdot\upsilon$$



Α.Δ.Ε.Τ $U + K = E_{\tau\alpha\lambda}$

$$\frac{1}{2}50x^2 + \frac{1}{2}m_{o\lambda}\upsilon^2 = \frac{1}{2}50\cdot 0,4^2 \Rightarrow$$

$$50x^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 = 50\cdot 0,16 \Rightarrow$$

$$50x^2 + 4\cdot\frac{3}{2} = 8 \Rightarrow$$

$$50x^2 = 2 \Rightarrow x = \pm\frac{1}{5} = \pm 0,2m$$

στην περίπτωσή μας $x=0,2m$ και $\upsilon = -\sqrt{3}/2$

άρα $$\frac{dW_{T\sigma\tau}}{dt} = -\frac{150}{4}\cdot 0,2\left(-\sqrt{\frac{3}{2}}\right) = +7,5\cdot\sqrt{\frac{3}{2}}\ J/s$$

ΘΕΜΑ Ε

Ε1) $\Sigma F_2 = 0 \Rightarrow T_2 = W_{2x} \Rightarrow T_2 = m_2 g \eta\mu 30 \Rightarrow T_2 = 4mg \cdot \frac{1}{2} \Rightarrow T_2 = 2mg$ (1)

τροχαλία $\Sigma \tau(0) = 0 \Rightarrow T_1 \cdot R - T_2 \cdot r = 0 \Rightarrow T_1 \cdot 2r = T_2 r \Rightarrow T_1 = \frac{T_2}{2}$

$\Rightarrow \boxed{T_1 = mg}$

$\Sigma F_1 = 0 \Rightarrow T_1 = W_1 + F_{ελ} \Rightarrow mg = mg + F_{ελ}$

$\Rightarrow K\Delta\ell_0 = 0 \Rightarrow \boxed{\Delta\ell_0 = 0}$

Ε2) $\Sigma F_1 = m_1 a$ Θετική φορά πάνω

$-W_1 - F_{ελ} + T_1 = m_1 a_1$

$T_1 = M_1 a + W_1 + F_{ελ}$

$T_1 = 1(-100x) + M_1 g + k \cdot x$

$T_1 = -100x + 150x + 10$

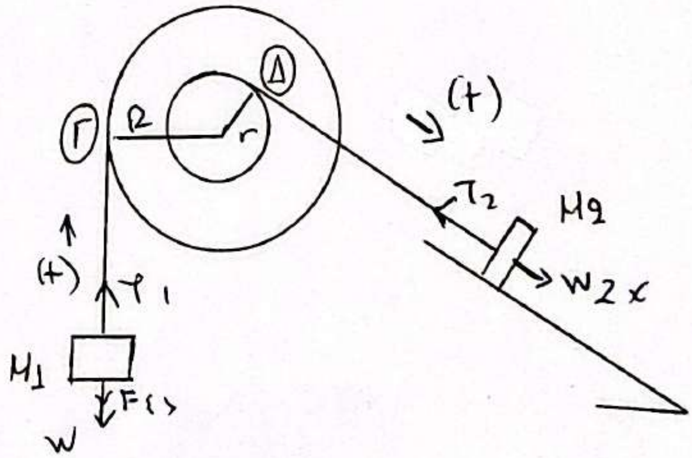
$\boxed{T_1 = 10 + 50x}$

$\Sigma F_2 = m_2 a$ Θετική φορά κάτω

$-T_2 + W_{2x} = M_2 a_2$

$W_{2x} - M_2 a_2 = T_2$

$\boxed{T_2 = 20 - 4a_2}$



$|a_1| = a_{\varepsilon\pi\iota\tau(\Gamma)} = a_\gamma \cdot R$, $|a_2| = a_{\varepsilon\pi\iota\tau(\Delta)} = a_\gamma \cdot r$ $\Rightarrow \frac{|a_1|}{|a_2|} = 2$

άρα $a_2 = \frac{a_1}{2} = -50x$

οπότε $20 - 4(-50x) = T_2$

$\boxed{T_2 = 20 + 200x}$

Ε3) Αν τα νήματα χαλαρώσουν $T_1 = 0$, $T_2 = 0$

$10 + 50x = 0$ και $20 + 200x = 0$

$x = -0,2m$ $x = -0,1m$

Άρα $|x_{max}| = 0,1m$

και $A_{max} = 0,1m$

Ε4) (α) $x_2 = \frac{\sqrt{3}\cdot 0,1}{4}$ $\qquad x_1 = \frac{\sqrt{3}\cdot 0,1}{2}$

ΔΕ.Τ $\qquad \frac{1}{2}D_1 x_1^2 + \frac{1}{2}M_1 u_1^2 = \frac{1}{2}DA^2$

$M_1\omega^2\left(\frac{\sqrt{3}\cdot 0,1}{2}\right)^2 + u_1^2 = M_1\omega^2\cdot 0,1^2$

$\frac{3}{4} + u_1^2 = 1 \Rightarrow u_1^2 = \frac{1}{4}$, $u_1 = \pm\frac{1}{2}\ m/s$

πρώτη φορά $\quad u<0$ άρα $u = -\frac{1}{2}\ m/s$

$a_\kappa = \frac{u^2}{R} = 5\ m/s^2$

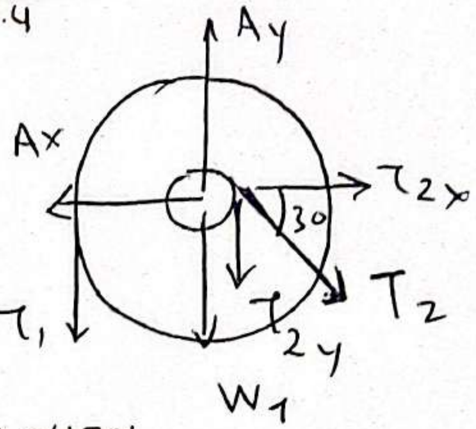
$a_{\varepsilon\pi} = |a_1| = \left|100\frac{\sqrt{3}\cdot 0,1}{2}\right| = 5\sqrt{3}\ m/s^2$

$a_{o\lambda} = \sqrt{a_\kappa^2 + a_\varepsilon^2} = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{5^2\cdot 4}$

$a_{o\lambda} = 5\cdot 2 = 10\ m/s^2$

Ε4) $T_1 = 10 + 50\left(\frac{0,1}{2}\right) = 12,5N$

(β) $T_2 = 20 + 200\frac{0,1}{2} = 30N$



$A_x = T_2\sigma\upsilon\nu 30 = 30\frac{\sqrt{3}}{2} = 15\sqrt{3}N$

$A_y = T_2\eta\mu 30 + T_1 + w_T = 15 + 12,5 + 17,5 = 45N$

$A = \sqrt{45^2 + (15\sqrt{3})^2}$

$= \sqrt{15^2\cdot 9 + 15^2\cdot 3}$

$= \sqrt{15^2\cdot 12} = 15\cdot 4\sqrt{3} = 60\sqrt{3}N$

Ε5) $\frac{dT_2}{dt} = \frac{d(20+200x)}{dt} = 200u = 200(-1/2) = -100\ N/s$